Exercise 8.4.5

Nicolai Siim Larsen

02407 Stochastic Processes

We consider a geometric Brownian motion $\{X_t\}_{t\geq 0}$, which describes the spot price of a stock for a certain company. We are given the drift and variance parameters of the geometric Brownian motion, specifically $\alpha = -0.1$ and $\sigma^2 = 4$.

An investor buys a share of this stock at the current spot price of \$100. We denote this as $X_0 = 100$. We seek the probability that the investor will profit from the investment, i.e. that the stock price reaches \$110 before it reaches \$95. To this end, we will apply Theorem 8.3.

We therefore define the random variable T as on p. 426,

$$T = \inf\left\{t \geq 0 : \frac{X_t}{X_0} \in \left\{\frac{95}{100}, \frac{110}{100}\right\}\right\}.$$

Theorem 8.3 then yields that

$$\begin{aligned}
\mathbb{P}\left(\frac{X_T}{X_0} = \frac{110}{100}\right) &= \frac{1 - (95/100)^{1-2\alpha/\sigma^2}}{(110/100)^{1-2\alpha/\sigma^2} - (95/100)^{1-2\alpha/\sigma^2}} \\
&= \frac{1 - (95/100)^{1-2(-0.1)/4}}{(110/100)^{1-2(-0.1)/4} - (95/100)^{1-2(-0.1)/4}} \\
&= 0.3325.
\end{aligned}$$